Language Issues in the Teaching and Learning of School Algebra

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Aim

To investigate the link between language and competence in algebra, highlight the problems experienced by pupils when learning algebra, and suggest some strategies for teachers.

Dimensions of this Case Study

The research was carried out from September 1997 to May 1998 with 55 pupils aged 14-16 and two teachers at Pent Valley School in Folkestone, Kent.

Summary of Findings for this Case Study

For Pupils:

- A significant number of pupils with low reading ages and poor problem-solving skills demonstrated competence with algebra.
- Common misconceptions occurred because pupils could not relate to teacher language which reflected a sophisticated understanding of algebra.
- Pupils found it difficult to describe and discuss the algebra they used and generally did not use the language used by their teachers.

For Teachers:

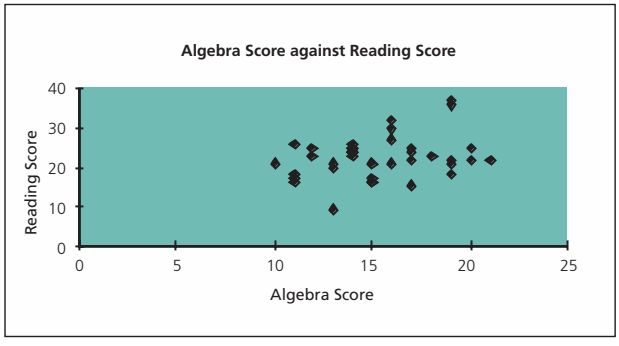
- It appeared that the main stress was on the learning of methods and building the confidence of the pupils.
- The skills and level of understanding associated with school algebra were accessible to many pupils where the teaching approach was appropriate.
- The vocabulary of algebra needed to be explicitly taught, practised and revised.
- Confidence was central to achievement in algebra. When planning the curriculum, emphasis needed to be on those areas where skills and understanding could develop together.

Introduction

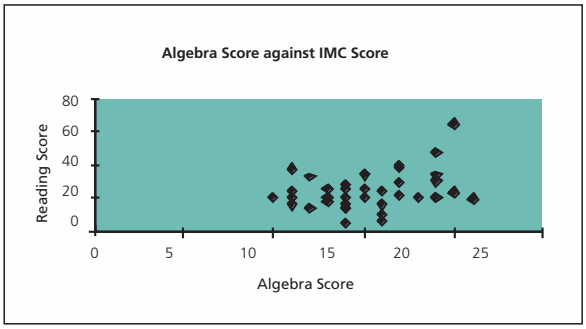
Ask any adult about school algebra and you tend to get a strong reaction. For many it brings back feelings of inadequacy, anxiety or blind panic; others had no great difficulty and actually admit to enjoying fiddling around with all those x's and y's. It is certainly not plausible to characterise the two groups as clever and stupid. Many of those who found school algebra so confusing were, and are, highly intelligent people who went on to excel in many different fields. This research looks at two aspects of this conundrum. Do most pupils who are good at algebra have other specific skills, such as good language abilities or good problem-solving skills? What can we as teachers do to help pupils improve their skills and levels of understanding in algebra?

Who's Good at Algebra?

Forty-five pupils aged 14-15 were tested on a written algebra paper which took them about 45 minutes. These same pupils sat the 'Intermediate Maths Challenge' problem-solving test and finally their reading ages were calculated. The results would show whether ability at algebra was related to language or mathematical problem-solving skills.



As the diagram shows, there is no strong relationship between the two scores. Pupils who did well on the algebra test (scored 17 or over) had a wide range of reading age scores (from 15 to 37, equivalent to reading ages from 11 yrs. 8 mths. to adult). It is a similar story for the problem-solving IMC scores. Those who did well on the algebra test scored between 20 and 65 on the IMC test.



The most significant aspect of these results relates to the teacher's expectation of how a class is likely to perform. We recognise that algebra is a difficult area and probably expect those pupils who are generally weak academically to struggle. Although the sample was from the upper end of the ability range, this research suggests that such a view is unfounded.

Where do Pupils Go Wrong?

"I don't mind algebra now because I can do it." This was a very typical comment from a 16-year-old pupil who described how only two years previously she had hated maths because she had not understood the algebra. The analysis of the test results together with observation of lessons and interviews with pupils provided a clearer picture of some of the problems which pupils experience:

- Forming algebraic expressions was easier when there was a diagram to help.
- There was confusion when organising expressions which contained numbers as well as algebraic terms.
- Many pupils were convinced that different letters cannot take the same numerical value.
- Weaker pupils thought that the alphabetical order of the letters was important.
- Pupils found it easier to interpret and use formulae than abstract equations in two unknowns.
- There was a widespread assumption that every equation poses the problem of finding an unknown value.

Teaching Algebra

The teachers involved in the research were interviewed and the pupil data examined in detail. They were convinced that the algebra component is the most difficult to teach – and the one they are least satisfied with in terms of delivery. Many pupils agreed with them. Even the brightest 16-year-olds admitted to confusion and frustration when they first started studying formal algebra.

The problem has several sources. The National Curriculum is widely recognised as being weak on algebra with the Mathematical Working Group of the Royal Society recommending a complete restructuring. These teachers felt that in many cases the order and the emphasis was wrong. Many examples were cited: "We teach simple manipulation early because its easy to get right, but often the pupils don't really know what they are doing." Certainly the pupils interviewed found it very difficult to explain what terms meant or the different uses of algebra. "I suppose it must be useful, but I don't know why letters are used so much," was one pupil's view. Many have stuck with very simplistic notions which they acquired quite early and then, when faced with more complex problems, adopt a spontaneous, common-sense approach which often does not reflect what they have been taught.

The teachers felt that they needed to prepare more carefully for algebra-based lessons because textbooks were not always coherent and because once lost, it was not easy to get pupils back on track. Overall it appeared that teachers stressed the learning of methods and tried to build pupils' confidence rather than finding ways to develop understanding.

The Implications

The language used when teaching algebra seemed to be crucial to our pupils' understanding but we generally did not make a conscious effort to teach this language and pupils certainly did not reproduce it spontaneously. The assumption that pupils pick it up as they go along is not backed up by this research. However, algebra is not as inaccessible as we tended to believe and pupils with a wide range of language and problem-solving skills achieved good results.

The misunderstandings which did occur demonstrated an over-emphasis on routine skills at the expense of the conceptual side. Of course, pupils need lots of practice; learning the procedures described by algebra is vital to success. However, this research seems to show that we are not taking the opportunities explicitly to teach the language pupils will need nor to develop their understanding. Textbook explanations or indeed the National Curriculum may not provide much assistance. The more reliable approach seems likely to be teachers sharing experiences and thinking their strategies through as carefully as a full timetable permits.

Discussion Points

Some specific do's and don'ts which led to useful discussion are given below. They are tentative suggestions in no particular order; further research into exactly how teachers go about their business in this area would be extremely valuable.

- Teach, revise and test the vocabulary which is introduced.
- Be explicit about when the understanding of 'variable' is important and when it is not.
- Do lots of simple formula work with diagrams before going on to abstract expressions.
- Encourage testing by substitution using fractions, decimals and negative numbers.
- Use formulae for sequences to discuss the values 'n' can take.
- Don't use codes when introducing algebra (e.g. a=1, b=2, etc.).
- Don't try to make too many connections at once, e.g. functions, equations, co-ordinates and graphs.
- Decide whether the algebra is being used to describe a procedure or to develop mathematical structure. Pupils are likely to be more confident with the former.

Methods

In addition to the quantitative tests described earlier, data was collected from lesson observation and interviews with pupils and teachers. During the lesson observations the algebra-specific language used by the teacher was noted. After the lessons a number of pupils were asked prepared questions which tested their understanding of the content of the lesson and prompted the target language. Further data was gathered from structured interviews with pupils who were not involved in the quantitative study.

Analysis and interpretation of the pupil data formed the basis of the extended, guided interviews with the teachers involved. Their comments together with the classroom observations led to the recommendations for teaching strategies.

Further Reading

Booth, L.R. Algebra: *Children's Strategies and Errors*, NFER-Nelson (1984)

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