Developing a need for algebra

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> Aim

To investigate whether a secondary mathematics classroom culture can be established in which students develop a need for algebra, using it themselves without prompting, having gained a sense of its power, and, if so, what factors led to this.

> Dimensions of this Case Study

This research was focused on one class of twenty-seven 11 and 12 year-old students and took place over the period of their first term in secondary school.

> Summary of Findings for this Case Study

The findings of this study show that it was possible to establish a classroom culture in which students generated a need for algebra, in a mixed ability year 7 group in their first term at secondary school. Evidence included:

- at the end of 15 weeks, over half of the students in the class had moved from never having met algebra before to being able to use algebraic symbols to express their own ideas;
- students asked with increasing regularity in problem-solving situations: 'Can we do this for N?';
- students showed understanding of the meaning of algebraic statements in different contexts and appreciated when the statements were useful or not within those contexts;
- students developed skill in algebraic manipulations as they made sense of algebraic ideas within the context of the activities chosen;
- it has not been necessary to drill the students in techniques;
- when the first coursework task was presented to these students they were able to structure their own approach to the problem in a way similar to that required in GCSE coursework.

Factors in establishing such a classroom culture successfully were:

- developing a wider classroom culture to do with 'becoming a mathematician';
- the teacher commenting *on* and highlighting the mathematical behaviour of students whenever it was observed;
- the choice of activities and teaching strategies used within those activities e.g. the use of common boards;
- an emphasis on students writing both in the act of doing mathematics and in reflection on what they have learnt.

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Background

In two recent national studies algebra was identified as: a key area in facilitating a smooth transition between GCSE and higher education (Winter, et al, 1997) and a topic that, within the framework of the National Curriculum, *"needs to be expanded and elucidated - indeed rethought"* (Sutherland, R., 1997, p.iii). A further conclusion was that *"more research is needed to understand the relationship between what algebra is taught and what is learned"* (ibid).

About the project

In standardised baseline tests at the start of term the group studied was found to be heavily skewed towards low attainment. Five students in the group scored at or above the national average for their age. The same number of students registered the lowest score possible on the test. The group was therefore of mixed attainment and included two students with a Statement of Special Educational Needs for their learning difficulties.

A classroom culture was established as described earlier. I put forward over-arching questions or principles that allowed the students to generate their own work. Self-checking mechanisms were put in place so that I did not have to be the arbiter of whether students were right or wrong. For example, a 'questions board' and a 'theories board' were set up so that students' work and ideas could be reviewed by their peers and stimulate class discussion. I intervened as necessary. Students were encouraged to write down what they did. By using video, teaching strategies could be monitored, allowing me to reflect on my own practice.

Definition of algebra

This study was based on Sutherland's (1997) proposed definition of algebraic activity as involving:

"(i) Generational activities - discovering algebraic expressions and equations;

(ii) Transformational rule-based activities - manipulating and simplifying algebraic expressions, solving equations, studying equivalence and form" (adapted from Kieran, quoted in Sutherland, 1997, p.28); and

"(iii) Global, meta-level activities which involve: awareness of mathematical structure, awareness of constraints of the problem situation, anticipation and working backwards, problem-solving, explaining and justifying" (Kieran, quoted in Sutherland, 1997, p.12).

I was initially interested to see if I could find evidence of the students using any or all of these aspects. The most problematic in terms of recognising these was (iii), but this was also the aspect of algebra I was most interested in promoting amongst the students. I took the essence of this aspect to be an awareness of what you were doing, a stepping back from the process you were engaged in, implying the ability to make choices about what to do next. Taken in this third sense, algebraic activity may be seen as underpinning the whole of mathematics.

Results: students finding a 'need' for algebra

The overwhelming evidence from this project showed that students did develop a need for algebra within the culture of the year 7 classroom. The students reported that before arriving at secondary school they had either little or no exposure to algebra nor to ideas of proof. By the end of one term several students demonstrated the ability to use algebraic proof as a way of answering a question which they themselves had posed within a problem situation. Over half the students showed evidence of using algebra to express their ideas. Across the attainment range students displayed an appreciation of two different uses of algebra:

- to prove a result which they believed to be true; and
- to show 'the workings' of a problem i.e. the use of algebra to express a rule.

In this way students showed an understanding of the meaning of algebraic statements in different contexts. Finding a rule or proof was not viewed by students as the end-point of problems but as a prompt for further activity.

The example below of using algebra for proof is from the book of a student of average attainment relative to the class. She was given this number trick:

think of a number; add on one more than that number; add 9; divide by 2; take away your original number.

	N
(+N+1)	2N+1
(+9)	2N+10
(÷2)	1N+5
(-N)	5
it will always end up with 5	

There is evidence here that the student naturally performed algebraic manipulations. She had had no drilling in techniques and yet within the context of the problem she was able to perform operations which are often seen as complex and difficult to teach. There was no explicit invitation to use algebra. The student had her own question: 'will it always end up 5?' and recognised for herself the power of an algebraic approach to answer it.

How was the classroom culture established?

The following four strategies were key factors in the setting up of a classroom culture in which students asked their own questions and hence found a need for algebra.

1) Giving the students a purpose for the year of 'becoming a mathematician'.

In the first lesson of the year I told the students that this year was about 'becoming a mathematician' and that this meant:

- thinking for yourself;
- noticing what you are doing;
- asking why things work;
- being organised; and
- looking for pattern.

The effect of this was to support the students in being aware of what they did in mathematics lessons by allowing them and me, the teacher, to question whether something they did was mathematical or not.

2) Highlighting and commenting on examples of mathematical behaviour, creating labels, (for students and teacher).

In every lesson I aimed, as much as possible, to highlight to students examples of when they or someone else was 'being a mathematician'. For example, if a student decided to approach a problem in a systematic manner I commented on this; e.g.: "That's an excellent example of getting organised, which is part of thinking mathematically". Over time, phrases such as 'getting organised' became labels that students used both in their writing and in talking about what they were doing. 'Getting organised' or 'asking why' became what students did in mathematics lessons. These labels were part of the culture of the group. Some labels originated from the students e.g. that being a mathematician means: 'it's okay to make mistakes'; and that you: 'share your problems with other people'.

3) The choice of activities and teaching strategies used within those activities, e.g. common boards.

The study took it as axiomatic that if students were to have the opportunity to experience 'getting organised', for example, then there must be the space and opportunity for them to have *not* been organised. This had some implications for the ways in which classroom activities were structured. The activities were accessible to all and capable of being extended through question posing.

It was important for me to recognise when to introduce skills to the pupils and allow them time to practise them. The pupils were given the opportunity to work on these skills within the context in which they arose.

It was vital that mechanisms were in place for students to check their own and others' work. This was usually organised by the use of *common boards* on which students would either pin up things for comment (e.g. to get others to check, to

ask a question) or to record results. It was by the action of classifying such information that it became natural for students to ask their own questions.

4) Emphasising students writing.

From the first lesson of term I encouraged students to write down what they were doing, what question they were working on and what they noticed or found out. This was established as being part of 'becoming a mathematician'. The effect of this was to provoke in students further awareness of what they were doing, and help them to make links with what I took algebra to be about. One result was that when the first coursework task was presented to these students they were able to structure their own approach to the problem in a way similar to that required in GCSE coursework.

Conclusions

This study set out to answer two questions:

- was it possible to establish a school classroom culture in which students could find a need for algebra; and
- if so, how?

The answer to the first question was yes and a strong link was made between finding a need and asking one's own questions. The teaching strategies above offered a tentative answer to the second question. The notion of 'becoming a mathematician' was carefully thought through before it was offered to the class. What I was able to notice and comment on in the classroom, in terms of mathematical behaviours, was dependent on what I believed thinking mathematically to be about. So, although the behaviours that the students and I labelled and used were powerful influences on student learning, that was not a function of what the labels actually were. Anyone else attempting to establish a classroom culture along similar lines would no doubt come up with different labels to 'getting organised', etc. What seemed important was that I, and increasingly the students, had *some* behaviours we believed in and therefore were able to notice. Only in this way did behaviours such as 'getting organised' have the chance of becoming meaningful to the students who would then feel a need to use them.

Methods

Evidence from the classroom was collected by the use of: video recordings, observation notes and post-lesson write-ups. Semi-structured interviews were audio-recorded with three pairs of students at the start and the end of the first term.

Validation

After collecting the data from the first term I worked with my Higher Education steerer, another teacher educator and a small group of secondary mathematics teachers on implementing the findings of this project in their own classrooms. There were some striking similarities in terms of observed effects on students' motivation and learning. I wish to thank: Laurinda Brown, Jenny Brooks, Fiona Clemes, Alastair McLeod, Chris Smy, Ali Taylor and Jan Winter for their involvement, as well as Kingsfield School for supporting this research.

Further reading

Sutherland, R. 1997, *Teaching and Learning Algebra pre-19*, London: RS/JMC

Winter, J., Brown, L., Sutherland, R. 1997, *Curriculum Materials to Support Courses Bridging the Gap Between GCSE and A Level Mathematics,* London: Schools Curriculum and Assessment Authority

Articles and papers connected with this project have been contributed to the following:

Proceedings of the Day Conference of the British Society for Research into Learning Mathematics, Leeds, 1998, and Open University, 1999.

Mathematics Teaching 168, Journal of the Association of Teachers of Mathematics.

Proceedings of the Twenty-third Annual Conference of the International Group for the Psychology of Mathematics Education, Haifa, 1999.

The number trick shown above and several other ideas used in this project were taken from 'Median' worksheets. These materials are developed and produced by Don Steward, Maths Education Development Initiative and Advice Network, c/o Harlescott School, Shrewsbury, SY1 4LL. Tel: 01743 450404.

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