Research for Teachers

Collaborative mathematics

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How can collaboration and discussion in class improve the teaching and learning of mathematics?
Teachers and researchers have been concerned with how best to help pupils overcome their difficulties with mathematics for many years. This Research for Teachers (RfT) summary* describes an approach that has helped some teachers address these difficulties. The project used a student-centred, collaborative and discussion based method for learning, with some positive results.
The key questions which the research set out to answer were:

- How can we design teaching using lessons from other research, so mathematics learning becomes more effective?
- What effect do student-centred and collaborative learning approaches have on student learning and attitudes to learning, and on teachers' beliefs and practices?
- What tools can be used to encourage collaborative learning in mathematics classrooms?

The researcher involved in the project designed a series of algebra lessons to be used with low attaining GCSE students. The design phase took two years, using observation of four teachers. The second phase involved the new lessons being implemented in 44 different classes over one year. The lessons were designed to promote discussion as a route to learning, using activities which encouraged reasoning, improved understanding of concepts, and collaboration. The teachers attended a series of workshops to introduce them to the materials and methods which they then put into practice.
The research concluded that student-centred, collaborative and discussion based approaches to learning were more effective than more traditional transmission methods, especially in the development of conceptual understanding of mathematics. Specific impacts included:

- improved pupil scores in algebra tests;
- increases in pupil motivation and reduction in anxiety around mathematic; and
- more student-centred practices used by teachers.

In the lessons designed by the researcher, traditional practices such as asking students to practise calculations repetitively with the same methods gave way to discussions about concepts and differing methods. Features of practice which were shown to have a particularly beneficial effect on student performance and learning were:

- viewing mistakes as positive - identifying them and using them in subsequent discussions;
- allowing students to develop and justify their own varied methods; and
- encouraging students to set each other problems to solve.

The research also concluded that teachers' existing beliefs about teaching and learning are critical in determining teachers' practice, and in determining the extent to which they can be encouraged to experiment with new practices.

The research focused on students who were retaking GCSE mathematics at 44 FE colleges in England, but the approach is applicable to secondary students of all ages and phases. Indeed the discussion and reflection approaches suggested by the researchers have already been further developed for younger learners in schools.

This summary extends our earlier summary 'Raising achievement through group work' which examined evaluations of classroom approaches which helped primary pupils learn to work collaboratively and improve their capacity to think into a specific curriculum area.

*The RfT is based on the following publication:

Swan, M. (2006) Collaborative learning in mathematics: A challenge to our beliefs and practices. National Research and Development Centre for adult literacy and numeracy (NRDC) and the National Institute of Adult Continuing Education (NIACE). A CD of resources and video clips is available with the book.

Back to top

Overview

Learning mathematics through collaboration and discussion

How powerful is teaching and learning which emphasises collaboration and discussion by pupils? This RfT explores how mathematics lessons can be designed to stimulate such a classroom culture.

Why is the issue important?

Despite schools and teachers constantly looking for ways to improve the teaching and learning of mathematics, there remain groups of students who struggle to achieve in mathematics and who find particular concepts difficult to grasp. This research provides evidence that altering the style of teaching and the classroom culture towards collaboration, discussion and student-centred practices can be beneficial to the learning of mathematics.

What did the research find out?

The study concluded that student-centred, collaborative and discussion based approaches to learning were more effective than traditional transmission methods in which the teacher explains a topic and asks students to repetitively complete exercises. The lessons were particularly successful at assisting the development of conceptual understanding of mathematics.

Specific impacts included:

- improved pupil scores in algebra tests
- increases in pupil motivation and reduction in anxiety around mathematics
- more student-centred practices used by teachers

After the project those students who experienced the discussion based lessons were more active in their learning and reported being more actively engaged with people (ie. classmates and teachers) and with tasks.

The new lessons stimulated different kinds of classroom talk, including:

- students giving more reasoned explanations
- using conflict or mistakes to improve understanding
- greater participation in whole class discussions

Use of the new lessons also stimulated a shift in the beliefs of teachers. After the project may more teachers

supported a more 'connectionist' style of teaching which is characterised by teachers emphasising links between topics, valuing students' own methods and encouraging learning through dialogue between teacher and student.

How was this achieved?
The teachers involved took part in a one year professional development programme including several workshops which challenged their beliefs and practices, allowed them to test out the use of discussion and collaboration in teaching and learning and gave them the resources required to deliver a series of new algebra lessons to try out on their GCSE re-take mathematics classes.

In the new lessons traditional practices, including students being asked to practise calculations repetitively with the same methods, gave way to discussions about concepts and differing methods. Features of practice which were shown to have a particularly beneficial effect on student performance and learning were:

- viewing mistakes as positive - identifying them and using them in subsequent discussions
- allowing students to develop and justify their own varied methods, and
- encouraging students to set each other problems to solve.

The classroom materials made use of three general types of activity:

- evaluating the validity of statements - is a statement 'always, sometimes, or never true?'
- interpreting and classifying multiple representations of mathematical objects, and
- creating and solving new problems.

The research based the design of the new lessons on several key principles, including:

- lessons were conducted in supportive social contexts, with plenty of opportunities for feedback to students
- lessons consisted of rich, challenging tasks and questions
- teaching emphasised methods and reasons rather than answers, and
- students created links between mathematical topics.

How was the research designed to be trustworthy?
The research was rigorous, using a large sample and a variety of tools to measure impact of this set of lessons. 64 teachers from 44 FE colleges in England took part in the project. They were all teaching 16-21 year old students who were re-taking mathematics GCSE, having previously gained a grade D or below. The teachers were provided with a guide and resources for ten lessons and a diary to record the algebra lessons, track student attendance and progress throughout the project. Many of the lessons were observed by the researcher.

Several elements were measured before and after the project, using well tested tools, several developed specifically for this project. These measures included:

- students' performance in algebra tests
- students' attitude to learning mathematics, confidence and motivation
- students' views of differing teaching style
- teachers' beliefs, and
- teachers' practices.

What are the implications?
The research showed that:

- students are engaged and enthused by mathematics when they are given opportunities for active learning through discussion with peers and when they are encouraged to explain their thinking.
- learning of mathematics can be enhanced by encouraging students' mistakes to be seen as learning points, and by students setting each other problems to solve.
- teachers learned best in the same way as students, through discussion and reflection.

What do the case studies illustrate?

The case studies included in the RfT show, for example, how:

- a school incorporated discussion and collaboration into its new mathematics curriculum after trialling new practice and consulting key stage 4 students.
- retention rates improved when one school decided to make its 'A' level teaching more active and collaborative.
- the quality and depth of student mathematics talk in pairs improved in one school when the talk was modelled by teachers.

Back to top

Study

How can collaboration and discussion in class improve the teaching and learning of mathematics?

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What effect did the approach have on student performance?

Discussion lessons and gains in performance

Students were given an algebra test before and after the project. In general, greater learning gains were made by students who had experienced more of the new lessons. (For the remainder of this summary, the new lessons designed by the researcher will be described as the 'discussion lessons').

Of those students who experienced many of the discussion lessons, 77% improved their pre-test score and 18% regressed. By contrast, of those students who experienced none of the discussion based lessons, 61% improved their score and 35% regressed. The student group was a challenging one, comprising re-take students who had previously gained a D grade or lower at GCSE. The researcher concluded that this group typically has great difficulty with algebraic concepts, so even after experiencing the discussion lessons a small percentage of students had still regressed.

Analysis of the number of questions answered by students in the tests indicated that the improvements were mainly due to a reduction in the number of errors made. The gains were not simply a result of students having an increased motivation to attempt to answer more of the questions.

Teaching styles and gains in performance

The researchers also looked at the impact of the style of teaching that students experienced on student performance. This focussed not just on how many of the new discussion lessons were taught, but how many of them were taught in the style which was intended (i.e. discussion-based and student-centred). At the end of the project the students themselves rated the extent to which their algebra lessons had been student-centred.

When many of the discussion lessons were taught and a more student-centred approach was adopted by the teacher, then 83% of students improved their performance and only 13% regressed. When no discussion lessons were used and more teacher-centred approaches had been adopted then 50% of students improved but 50% regressed.

So it appeared that both the number of discussion lessons taught and the predominant style of the teaching were both related to improved algebra learning outcomes.

In what areas were the greatest performance gains made?

The researchers also looked in detail at individual areas of the algebra tests to identify where the biggest gains had been made. The greatest gains were made in:

- substitution;
- constructing algebraic expressions and equations;
- solving simple equations; and
- handling inequations - a statement that two objects or expressions are not the same, or do not represent the same value. This relation is written with a crossed out equals sign, like $x \neq y$.

'Substitution' is one of the key areas of all algebra courses, requiring students to interpret symbols and carry out operations in the correct order. Much greater gains were made in lessons which placed emphasis on students articulating the meaning of an expression verbally in a group, rather than simply being asked to learn the rules and practice individually.

In the area of 'constructing equations' the students who took part in discussion lessons improved their performance whereas those who did not actually regressed. The discussion lessons encouraged students to talk through the meaning of different elements of an equation, rather than simply use letters and symbols without context.

Discussion and reflection activities offered many opportunities for students to construct and interpret their own expressions and equations and this led to further improvements in learning.

The researcher concluded that it is a challenge to assist lower attaining students to achieve learning gains and that some students continued to find algebra a difficult topic, but that the discussion based lessons did lead to measurable improvements in the performance of a greater number of students.

What effect did the approach have on student learning?
The use of the student-centred, discussion based algebra lessons had an impact on student learning in four main areas.

Students' preferred ways of working
One of the main intentions of introducing the discussion-based approach to learning was to encourage students to move from passive learning strategies to more active ones. Active strategies involve students:

- cooperating with peers;
- taking the initiative;
- facing challenges;
- being creative; and
- showing determination.

With passive strategies, students tend to:

- work alone;
- avoid challenge; and
- memorise methods and results provided by others.

The resources offered to the teachers encouraged them to promote more student-centred, active ways of learning and the results suggested that these were taken up by the students themselves.

After the project those students who experienced the discussion based lessons were more active in their learning and reported being more actively engaged with people (i.e. classmates and teachers) and with tasks. Those who had received none of the lessons became increasingly passive. Students also indicated that they

were more likely to ask the teacher questions after they had experienced the discussion lessons. Practitioners may like to read a case study that describes how one school introduced a more active learning and discussion-based curriculum which responded to the preferences of the students themselves.

The researcher also concluded that in most FE retake classes, as students approached their GCSE examinations, the use of active approaches to learning declined. The use of discussion activities in the project appeared to stop, if not reverse, this reduction.

Confidence, anxiety and motivation of students
The researcher noted from previous research that teachers who adopt more 'transmission styles' of teaching, in which knowledge is 'passed' from teacher to student, lower the self esteem of students who prefer more active and creative methods. The activities designed for this research project reversed that process.

The researcher measured the confidence, anxiety and motivation of students before and after the project. The students who had not used any of the discussion activities showed a decline in their overall confidence in their own ability to do mathematics and in their motivation towards the subject, while their anxiety about algebra as a topic increased over the year. In contrast there were no changes in the confidence, anxiety or motivation of the students who had experienced the lessons. More student-centred approaches therefore prevented a general decline in confidence and motivation of a group of students who had previously under-achieved in the subject.

One teacher's comment reflected the potential impact of a change in teaching and learning styles whilst cautioning against expecting too much too quickly:

By the end of a lesson they felt that they had really understood something... they were proud of their work... everybody commented on how much discussion had taken place... however some misconceptions started to appear later... many years of thinking wrongly could not be undone in a single lesson.

Engagement
There were indications that experiencing the new lessons helped students to engage better with the subject. For example, one teacher who taught only a few of the lessons reported:

I also noticed that during these lessons the whole of the two hours was spent having heated discussions about mathematics, as opposed to the students' social lives! There was an element of healthy competition as to who could create and who could solve the most complicated equation.

You may like to read a case study in which a school saw an increase in students taking and completing mathematics courses in Year 12, when the A-level lessons had been made less 'lecture like' and more student centred.

Students' awareness of their own learning
The researchers looked at the self-efficacy of students - i.e. their own perception of their capacity to do specific tasks. Those students who experienced more of the discussion and reflection lessons gained more confidence in their capacity to answer questions correctly. The researchers also found that these gains in confidence in their own ability were matched with actual gains in performance on the algebra tests. So the use of discussion lessons led to students having greater realism about which questions they were capable of answering successfully.

What kind of classroom talk did the lessons produce?
The lessons were intended to stimulate more open discussion and collaboration, with students becoming more engaged with mathematical concepts and more confident to enter into conversation as a means to learning. There were many examples of ways this occurred within the lessons, including:

Students giving reasoned explanations

During the project more and more student-student discussions became evident in some of the classrooms; the more intensive ones involved students giving reasoned explanations. The following transcript shows four students discussing the validity of the following statement:

In January, train fares went up by 20%. In August they went down by 20%. Fares are now at the same level that they started at.

Ant: Yes, £10 so it's 90 quid, no 20% so that's £80. 20% of 100 is 80... no 20.
Dan: Say the fare was 100 and it went up by 20%, that's 120.
Sally: Then it went back down so that's the same.
Andy: No, because 20% of 120 is more than 20% of 100. It will go down by more so it will be less. Are you with me?
Ant: Would it go down by more?
Andy: Yes because 20% of 120 is more than 20% of 100.
Ant: What is 20% of 120?
Dan: 96...
Andy: It will go down more so it will be less than 100.
Dan: It will go to 96.

In this class some of the students noticed and commented positively on the benefits of the new discursive style.

Highlighting and then resolving conflict
In another class students were asked to choose the correct equation to accompany a verbal situation:

On a trip, one adult is needed for every six children. a=number of adults, c= the number of children. Which of these is correct? a=6c, c=6a, c=a/6, a=c/6.

The teacher found that each answer had been chosen by at least one student. He used 'cognitive conflict' to engage their interest:

Watch this carefully, because this is an opportunity for you to really get to grips with it. Let's be honest. Out of the group here, we have got someone thinking that each of these is correct. They can't all be.

There followed a 20 minute class discussion on this one problem, followed by another 20 minutes on a second similar problem.

Encouraging greater participation in discussion
The 'cash till' activity in one of the lessons was intended to introduce simultaneous equations:

A cash till contains some £1 coins and come £5 notes. c= the number of coins, n= the number of notes. The fo llowing are true. 3n=c and 5n+c=80. Search for values that satisfy both equations.

Two of the classes observed had a similar and common difficulty, assuming that n could only take the value 5 because the note was worth £5. One teacher took more of a 'back seat', allowing students to discuss the p roblem and affirming and appearing to welcome their contributions:

Teacher: We are just looking at the number. Why are you putting in numbers?
Andy: It makes it easier.
Max: Just be 25 as you said.
Andy: I'm just trying to work out where the 80 comes from.
Michael: Is that the number of notes or the value?

Teacher: Good question. These letters are only numbers.
Michael: It doesn't tell you value, just the number. If there was a pound sign then it would be values, but there are just numbers there.
Teacher: Yes, it's only talking about numbers there.

The teacher then passed responsibility back to the students to work in pairs. When the class reconvened one student shared that he had found values that satisfied both equations, but the teacher continued to ask others to go to the front and take over the explanatory role while he acted as facilitator and 'devils advocate'. In one observed lesson the bell went before many issues had been resolved and the class left, arguing and debating among themselves.

By contrast, a different teacher took more control when leading the same lesson, directing the discussion from the front. The researcher observed that she became increasingly frustrated, feeling students would not accept what she said and she appeared to miss some useful and penetrating questions from students.

Practitioners may wish to read a case study in which a group of mathematics teachers improved the quality of student talk through modelling and providing a framework for this to happen.

What was the impact on the teachers who tried the new approach?
Before the project the 64 mathematics teachers in the sample tended to:

- use predominantly teacher-centred approaches;
- present mathematics in a predetermined, closed and heavily structured fashion; and
- emphasise routine skills to be practiced independently.

The teachers were asked to give ratings to the frequency with which they adopted 25 classroom practices. The top 13 most frequently used practices were all teacher-centred and included:

I tell the students which questions to tackle.

I teach the whole class at once.

I know exactly what the maths lesson will contain.

I tend to follow a text book closely.

Students use only the methods I teach them.

Students learn through doing exercises.

The teachers generally did not encourage the students to work collaboratively, show creativity or make decisions about what they learned. Lessons were typically full of carefully graded practical exercises, with topics presented sequentially from the beginning, even though students had studied many of the topics previously.

But by the end of the project the teachers involved reported that they were using more student-centred ways of working, with the following features occurring much more frequently in their lessons:

- collaboration and discussion;
- encouragement of identification of mistakes; and
- linking of and movement between different mathematical topics.

After the project the teachers also reported a change in their own priorities for the teaching and learning of mathematics. The priorities at the end of the project were the 'interpretation of concepts' and the 'development of strategies for problem solving', whereas at the beginning the top priority had been the 'fluency of recalling facts and performing routine skills'.

Student reports of changes to teachers' practice
Students generally confirmed the reports of the teachers. Those who had received more of the discussion lessons reported greater experience of student-centred teaching. The researcher pointed out that this could have been because those teachers who were already student-centred in their practice were naturally drawn to teaching more of these lessons. Either way, it is clear that students noticed a difference in the way that they were being taught. Students noted greater change when they were taught a relatively high number of the discussion lessons. Students who experienced a higher number of the discussion lessons rated two of the student-centred statements particularly highly:

The teacher expects us to learn through discussing our idea.
The teacher encourages us to make and discuss mistakes.

Students who experienced more of the discussion lessons also reported that their teachers had started to encourage the use of alternative methods, not just showing one way of answering a question.

Wider impact on practice
The ten discussion lessons only covered parts of the algebra curriculum. Some of the teachers recognised that lasting change required an expansion of the approach:

We feel that the next step would be to integrate each activity into a series of skills/topics rather than thinking of them as a one off lesson.

There was however some evidence of wider impact on individual and team practice as a result of the project. For example, one of the teachers introduced the approach to her team at a staff away day:

... I am very glad that I got my team involved alongside me. If I had done it first...they would have seen me as the 'expert' and we would have lost the teamwork.

Even those who found the discussion lessons challenging to use reported some impact, for example:

I have become more determined to make lessons interesting for students rather than being obsessed with trying to cover the whole syllabus.

What styles and principles of teaching and learning were promoted in the discussion lessons?
The group of teachers involved in the first of the two phases of the research used a 'diagnostic teaching' programme using 'conflict discussion' lessons. One of the main aims was to expose common learning obstacles which students faced and to motivate students to reformulate their own understanding of concepts.

There were three phases to the 'diagnostic teaching lesson':

- exploring students' existing understanding and methods - through tests and interviews prior to teaching;
- provoking and sharing 'cognitive conflicts' - by getting students to compare their responses with those of others, or by asking them to do the same task using a different method; and
- resolving and consolidating conflict - by discussing the new concepts and methods in groups, and then using them on other problems.

These lessons contained creative features which the teachers were not already using, including using sorting activities and activities in which students were invited to design their own examples.

The researcher then built on the lessons learned, designing the ten discussion lessons for the main group of teachers. This was done employing the following principles:

- lessons are conducted in supportive social contexts, with plenty of opportunities for feedback to students in addition to awarding of marks;
- lessons consist of rich, challenging tasks and questions;
- students are encouraged to make mistakes and learn from them;
- teaching emphasises methods and reasons rather than answers, with students encouraged to act as teacher whenever possible;
- students create links between mathematical topics;
- the purpose of each lesson should not be too broad and should be communicated clearly to students; and
- appropriate use should be made of technology.

The intention was to foster a collaborative culture in the classroom in which concepts could be discussed intensively and misconceptions identified and worked on. Practitioners may like to read a case study from our previous RfT on secondary school mathematics which describes the positive impact which classroom discussion and willingness to discuss mistakes can have on student performance.

The teaching activities designed for the project were not intended to remove completely the need for direct instruction, rather to limit its use. For example, in the lesson on creating and solving equations, the teachers began by demonstrating a method for creating an equation. Students were then asked to use this method to design their own equation. Traditional 'following a method' teaching was therefore still used, but in ways that allowed space for subsequent student creativity.

What materials and exercises did the research lessons use?
The teaching resources comprised:

- classroom materials for ten lessons of between one and two hours;
- teaching guidelines;
- illustrative video clips of the resources being used (this CD RoM is available with the book on which the RfT is based); and
- questions for reflection and discussion to encourage teachers to think more deeply about the issues involved.

The classroom materials made use of three general activity types:

- evaluating the validity of statements - is a statement 'always, sometimes, or never true?';
- interpreting and classifying multiple representations of mathematical objects; and
- creating and solving new problems.

The first set of activities provided a number of statements/solutions and students were asked to decide upon their validity, justify their decision with examples and counter-examples and give explanations. These activities covered some common misconceptions identified through observation and student interviews. The focus was on reasoning rather than simply obtaining answers.

For example:

$12a>12$ - if you multiply 12 by a number the answer will be greater than 12.

If you double the lengths of the sides of a 2-d shape, you double its perimeter.

The second set of activities used symbols, formulae, tables, graphs and verbal expressions. Card sorting

allowed these to be shared, interpreted, compared and classified. In noticing the 'sameness' or 'difference', students began to create and refine their understanding of concepts.

For example:

Students were given a large set of representations and had to match them - such as '2(n+6)' with 'add 6 to n then double the answer.'

The third set of activities invited students to create their own problems and examples for other students to solve. The designers and solvers worked together to identify difficulties, assisting each other and resolving difficulties collaboratively.

For example:

Make up questions that may be solved using only the numbers and information given below. Calculate your answers, then exchange questions with your neighbour:
My car holds 40 litres of petrol, its consumption is 40 miles per gallon, 1 litre is about 0.22 gallons, 1 litre costs about 56p.

Students were also given 'target questions' to tackle at the beginning and end of each lesson. These questions gave examples of what the lesson was intending to help them learn. By completing the target questions again at the end of the lesson, teachers and students could monitor whether the goals had been achieved.

The CD RoM provided to teachers gave three video extracts of each activity type and was used to convey what it felt like to try out discussion activities for the first time. They were not staged, including, for example, teachers making mathematical mistakes themselves. The CD RoM is available with the book which forms the basis of this RfT.

How important were teachers' existing beliefs and did they change?
The researcher looked at teachers' underlying beliefs about the teaching and learning of mathematics, how these beliefs influenced practice and whether teachers' beliefs changed over time.

Teachers' predominant beliefs were broadly separated into three categories by the researcher:

- 'Transmission'

This is the belief that knowledge and methods are best conveyed directly to students by the teacher, with more emphasis on teaching than learning. Teaching is followed systematically and students are typically blamed if they are unable to grasp what is being taught.

- 'Discovery'

This is the belief that places more value on students' own creativity, with all methods being acceptable so long as the answer is correct. Learning is seen as an individual activity but only happens for students who are 'ready' to discover things for themselves.

- 'Connectionist'

In this belief system teachers emphasise links between topics to help develop reasoning and proof. Students' own methods are valued, misunderstandings are seen as an important part of lessons, and crucially, learning occurs through dialogue between teacher and student. Connectionist teachers are much more likely to encourage students to discuss their ideas and use prior learning. You may like to read our earlier RfT about effective teachers of numeracy which describes the characteristics of connectionist teachers and the positive

impact this can have on the teaching of numeracy.

The researchers found that teachers who started with connectionist beliefs were more likely to have volunteered to be part of the research project and more likely to try more of the discussion lessons. Connectionist teachers were more likely to seek continuing professional development of the type provided by the project.

During the project there was a strong shift towards connectionist beliefs among the teachers (6 began the project with this belief system, 18 by the end). There was a corresponding shift by the teachers away from transmission beliefs (from 18 to 10) and discovery beliefs (from 7 to 3). Of the teachers who began with transmission beliefs, those who had taught more of the new lessons were the ones whose beliefs shifted most toward a connectionist orientation.

Some of the teachers who moved towards connectionist belief had already recognised that their previous teaching approaches were not working well and early on expressed a desire to change their practice. They appeared more willing to take risks and persist over several lessons. They were surprised and delighted by the change in the engagement and attitude of their students. For these teachers their practices changed first and their beliefs followed. Those who adopted connectionist beliefs all encouraged more discussion of mistakes in the classroom. You may like to read a case study that shows how the use of an enquiry based curriculum at one school led to changes in practice and ultimately changes in the beliefs of teachers about what constituted good practice.

Several of the teachers confirmed the confidence they held in their new beliefs, developed through practice, by holding meetings with colleagues to disseminate the ideas more widely or writing additional resources of their own using the same principles.

The eight teachers who remained predominantly believers in the transmission approach appeared to the researcher to be less reflective about their practice. The most extreme transmission orientated teachers interpreted their experiences in ways which reinforced their beliefs - for example, one teacher believed his class had been unwilling to take part in discussions, but the researcher had observed him tending to curtail discussion with his own interventions and instructions.

What factors stopped some teachers putting their beliefs into practice?
Before the project only three of the teachers claimed that they taught in ways which were entirely compatible with their own beliefs. For example one teacher said:

I do not believe in trying to get through the whole syllabus and as a result trying to pressurise students into working faster than they are able, but I do this.

The teachers sometimes worked in ways that were contrary to their beliefs about good teaching because they felt:

- concerned about covering the full syllabus (in spite of the little time that they had available to do this);
- under-resourced to meet the challenge of changing their approach, with insufficient preparation time or resources to cover a topic in a more creative way;
- they needed to meet external expectations and perceived influences of the 'system' on their practice (e.g. government policy or college management dictating how they should teach); and
- students' expectations were low.

Despite these perceptions the project resulted in changes to both practice and belief for many of the teachers.

An Ofsted inspector was invited to one of the workshops to alleviate the concerns of some teachers about the inspection implications of using a discussion-based method of learning. Subsequently, teachers who were

inspected while using the new discussion lessons all reported positive and encouraging feedback.

How did the workshops encourage teachers to try new practice and re-consider their beliefs?
The researcher recognised that teachers needed high levels of support in order to try out new ways of working. The four days of workshops spread over six months gave this support by being deliberately structured using the following principles:

- establish an informal, candid and non judgemental culture;
- illustrate the use of contrasting practices;
- ask teachers to 'suspend belief and act in new ways';
- encourage teachers to meet together and reflect on new experiences; and
- ask teachers to reflect on and recognise the growth of new beliefs.

The teachers were issued with workbooks which contained guidance and space for individual reflection on various area, including; the teaching of mathematics, common student misconceptions, use of discussion in lessons, activity types and designing lessons. The need for honesty and openness was stressed during the workshops and the researcher noted greater depth and honesty of discussion as the workshops progressed.

Each time activities for students were introduced at the workshops, the teachers carried out a similar activity at a higher level themselves, so that they could experience the process of learning themselves, for example:

Always, sometimes or never true?
The square of a prime number is always one more than a multiple of 24.
Quadrilaterals tessellate.

Teachers were encouraged to design and try out their own activities between the workshops and were given diaries to record progress and received telephone support from teacher mentors involved earlier in the project.

Workshops were structured to allow plenty of time for the teachers to reflect on issues, successes and challenges and to describe how students and colleagues had responded to their work. By the fourth workshop teachers were feeling comfortable enough with each other to share their thoughts and fears openly.

The researchers discovered that teachers' own beliefs about teaching and learning are often strongly held and hugely influential over practice in the classroom. Nonetheless there was evidence that these beliefs can change, most strikingly if teachers can be persuaded to 'suspend belief' and try out new practices.

One teacher who changed from transmission to connectionist belief reported:

I can't get them away from the board now - coming out and doing questions... one of the students used the expanding brackets using areas as a solution. It was really good considering we had only done three lessons in October and November. So retaining it in January was really brilliant.

Another teacher explained:

Lively discussion ensued. Lots of disagreement on making the equations..... I used a non directive approach and all students got involved. A lot of misconceptions came out.... This class was inspected and this was one lesson he was impressed by.

You may wish to read a case study that describes how teachers from six schools experienced collaborative CPD over one academic year which resulted in active and discussion orientated practice.

How was the evidence gathered and analysed?

Evidence was gathered and analysed in order to answer three research questions:

- How can we design teaching so that mathematics learning will become more effective?
- What are the effects of applying such principles to the design of teaching GCSE re-take classes in FE mathematics classrooms?
- What conceptual tools facilitate the construction of collaborative cultures in these classrooms?

The study consisted of two phases. During the first phase, lasting two years, the main researcher interviewed teachers and visited four classrooms to look at existing practices and monitor their effects. Baseline data were collected on approaches used by the teachers and common difficulties and misunderstandings by students. The researcher designed a collection of classroom discussion activities using the diagnostic teaching approach. These were then trialled and tested. Lessons were observed and recorded and students' levels of understanding were monitored through pre and post testing.

For phase two, the main part of the study, 64 teachers from 44 FE and sixth form colleges in England were involved. The researchers created a one-year professional development programme. Algebra was chosen as the topic for three reasons:

- It was considered fundamental to all mathematical work and one which lower attaining students find hard to grasp as a general concept.
- Algebra was a topic that had not shown much improvement during the first phase of the project.
- The teachers in phase one agreed that algebra is often taught in an unimaginative, transmission oriented way.

The course for teachers consisted of a two-day residential workshop held during October, to introduce the methods and resources. This was followed up with two one-day workshops in the following January and March in which teachers reported back on their experiences and reflected on implications for their practice having explored the use of discussion and reflection in their own classrooms.

Of the 36 teachers who attended the workshops, 14 taught 'many' of the 'discussion lessons' (between 7 and 14 lessons), 14 taught 'few' (between 3 and 6) and 8 taught a variable number but provided no post-test student data.

A group of 14 teachers from the same colleges/schools taught none of the lessons but were involved in the project as a control group, to help compare the student gains.

The teachers were provided with a guide and resources for ten lessons, along with a CD RoM which contained examples of the three types of activities.

Teachers were also given a diary to record the algebra lessons, track student attendance and progress throughout the project. Teachers were given telephone support from mentors between the workshops and several lessons were observed by the researchers.

Teachers' beliefs were measured by being asked to rank various statements about the nature of mathematics, learning and teaching.

Students were given algebra and attitude questionnaires to complete before and after the project. These gathered data on their confidence, performance with algebra, areas of difficulty, motivation, preferred ways of working and views of teachers' practices.

The students who took part in the study were aged 16-21 and most had previously gained a grade D or E at

GCSE mathematics.

Implications
Teachers might like to consider the following questions in making use of the findings of the study:

- The study found that students are engaged and enthused by mathematics when they are given opportunities for active learning through discussion with peers and when they are encouraged to explain their thinking. Could you plan your lessons to include more time for discussion of concepts and explanations by students?
- The study concluded that learning of mathematics can be enhanced by encouraging students' mistakes to be seen as learning points, and by students setting each other problems to solve. Could you model classroom conversation which highlights the potential for learning from mistakes and design lessons which regularly expect students to set each other problems?
- The study showed that trying out challenging new practices on a sustained basis with colleagues helped most teachers review and refine their beliefs. Are there areas of the curriculum and of practice where there is scope for significant change, where working together with colleagues and a specialist presents an opportunity to explore and enhance the learning culture of your department?

School leaders might like to consider the following implications:

- The study found that teachers generally responded well to encouragement to increase student discussion and collaboration in mathematics classes. Could you support your head of mathematics to introduce a policy on the use of 'classroom talk/discussion' which could be trialled for a sustained but fixed period and then assessed?
- The study showed that teachers learned best in the same way as students, through discussion and reflection. If you do not already do so, could you include structured discussion and reflection in the systems you use to track CPD in your school,, so that you can explore what feeds through effectively into student learning?

Gaps in the research
Gaps that are uncovered in a piece of research have a useful role in making sure that future research builds cumulatively on what is known. But research also needs to inform practice, so practitioners' interpretation of the gaps and your own follow-up questions and enquiry are crucial. We think the following kinds of studies would usefully supplement the findings of the summary:

- case studies of approaches by teachers who have encouraged discussion and collaboration in the teaching and learning of mathematics;
- studies which identify alternative, effective ways to deliver those parts of the mathematics curriculum other than algebra which are frequently taught in a transmission or 'chalk and talk' style;
- research which quantifies the impact of discussion and collaborative teaching and learning on the performance of different groups of students; and
- studies which take the learning from this research and test it out with different age groups of students.

What is your experience?
Do you have any evidence of the ways in which mathematics can be successfully delivered through discussion and collaborative means? We would be interested to hear about examples of effective approaches which we could perhaps feature in our case study section.

Your feedback
Have you found this study to be useful? Have you used any aspect of this research in your own classroom teaching practice? We would like to hear your feedback on this study. Click on the feedback link to share your views with us.
Back to top

Case Studies

Case studies

We have chosen five case studies, all conducted by teachers, which illustrate aspects of the findings of the study, covering the teaching and learning of mathematics with a collaborative discussion based focus:

- the first study shows how a school incorporated discussion and collaboration into its new mathematics curriculum after trialling new practice and consulting key Stage 4 students;
- the second study shows the striking impact on retention achieved when one school decided to make its A-level teaching more active and collaborative;
- the third study demonstrates how three teachers improved the quality and depth of student mathematics talk in pairs, by modelling that talk and planning good practice;
- the fourth case study is of how, in one mathematics department, teachers' beliefs in the importance of discussion were changed as a result of introducing enquiry based practice; and
- the fifth study describes how teachers from a group of schools used a collaborative CPD model to make mathematics more active and discussion orientated for students.

Case study 1: Involving students in planning a discussion based curriculum

Case study 1: Involving students in planning a discussion based curriculum
We chose this study because it shows how deliberate changes in teacher practice were introduced which followed students' preferred ways of working. The new focus included active learning and discussion based activities in the revamped mathematics curriculum.

What was the background and aims of the study?
The study took place in a mixed comprehensive school near Cambridge. The research arose from a perception in the mathematics department that the school needed to improve the motivation, engagement and mathematical experience of lower attaining students at Key Stage 4 and a desire from teachers to discover students' perception of what helped them to learn.

The aims of the study included to:

- identify the types of activity which encouraged deep learning;
- discover the type of activity that students believed best helped them to learn; and
- improve teaching and learning by use of collaborative planning and the sharing of resources.

What methods did the school use?
Faculty conference
The project was launched at a conference day for all mathematics teaching staff. The day included:

- discussion of beliefs and what constitutes rich tasks and deep learning;
- development of a model 'excellent maths lesson';
- planning the observation and feedback of lessons; and
- design of student questionnaires/focus groups.

A second faculty meeting was held some months later to feed back the results of the student survey and discuss the implications this had for teaching.

Use of collaborative planning and observation of lessons to encourage deep learning
The collaboratively planned research lessons were taught by one teacher and observed by at least one other member of staff. Feedback from observations was shared.

Teachers involved trialled and adapted various activities for different groups of students. Teachers began to focus on the intended learning outcome rather than the age or ability of the group as had previously been the case.

The activities used in the research lessons encouraged most or all of the following:

- collaboration;

- discussion;
- reasoning;
- generalisation; and
- practical activity.

Student survey and focus groups
A questionnaire was given to a random sample of 10% of students in Years 7-10. The questionnaire asked students what they thought of mathematics lessons generally and which activities they particularly liked or found useful. This was followed up by some student focus groups to gather more detail.

Most students were clear that active learning through discussion and collaboration in small groups without text books was their favoured method of working. Problem solving was frequently mentioned positively in the focus groups. Students thought teachers talked too much and also did not like being asked to work quietly. There were several revealing comments in this regard:

'I don't like it when the teacher does the work on the board - sometimes you just want to do it yourself.'

'It's when the work gets hard that you need talk.'

Student feedback also flagged up where some activities were over-used:

'Sometimes we get fed up of card sorts and sticking them down, we might only do it a bit in maths but we do a lot in other subjects as well. Some days you might do it in five different lessons - that's a lot of sticking!' (Year 8 student)

What were the outcomes from the study?
Through lesson observation and student feedback the study concluded that effective classroom activities contained characteristics including:

- simple and clear presentation with minimum text;
- encouragement of collaboration and discussion;
- problems that can be solved in several different ways;
- encouragement of generalisations; and
- offering challenge, extension and opportunities for feedback.

Changes to teaching practice which resulted from this study included:

- limiting teacher talk time to 10 minutes wherever possible;
- sparing use of text books;
- practising skills limited to 10 minute bursts; and
- using images and equipment for active learning where possible.

The quality and focus of discussions within the department shifted away from classroom management towards teaching and learning as a result of the research. Collaboration and discussion between teachers and learners was identified as the key to improving both the teaching and learning of mathematics.

Reference
Harding, K. (2008) Increasing student engagement and motivation at key stage 4 by promoting deeper learning in mathematics.
National Centre for Excellence in the Teaching of Mathematics grant final report

Case study 2: Improving engagement and retention by moving away from the text book

Case study 2: Improving engagement and retention by moving away from the text book
We chose this case study because it shows how one school greatly improved student retention rates by using

collaborative activities, focussing on active learning and providing more open-ended problems.

Mathematics teachers at a specialist technology college in North Tyneside were aware that their A-level lessons were less interactive than their GCSE lessons, being more 'lecture like' in form. The teachers wanted to move away from this way of working and to check the validity of this view with students.

The project aimed to:

- find out about students' experiences of A-level teaching in mathematics;
- develop new teaching resources for use with A-level classes;
- improve the experience of students studying A-level mathematics; and
- increase the uptake of mathematics in Year 12.

The research centred on two A-level classes of 20 students each. Staff developed new teaching resources for each module of the course, and tested them on both a Year 12 and 13 class.

What impact did the new resources have?
The main findings of the research were that:

- retention of students between Years 12 and 13 increased - 60% completed the course in 2008 compared to 25% in 2006;
- increased demand led to a 'Further Mathematics' course being offered in Years 12 and 13;
- collaboration between teachers improved;
- more students applied to study mathematics at university;
- student motivation and enjoyment increased; and
- student-teacher relationships became more purposeful.

How were the changes planed?
An external researcher gathered the opinions of A-level mathematics students. The key finding at this stage was that students preferred lessons in which they worked independently or in a group on tasks which did not involve answering set questions from a text.

Staff from the mathematics department then gathered existing teaching resources and began to develop them in line with the findings from the student research. The lead subject professional for the local authority led an inset training day to introduce some new teaching materials, observed and fed back on some lessons, and taught some demonstration lessons which were video taped and discussed.

What new resources and practices resulted from the project?
Teaching staff developed a number of game based resources, including pairs games and sequencing activities. The new resources were used at least once per fortnight and their effectiveness discussed in staff meetings.

An overnight 'revision retreat' was also organised for Year 12 students in the lead up to their second modular exam in June. Active learning through activities such as floor jigsaws and treasure hunts were used to assist the revision. During the second year, the revision retreat was extended to include more of the A-level course and was moved to a larger venue to cater for increased numbers.

A library of useful resources was set up within the mathematics department and the project led to improved teaching practice in Key Stages 3 and 4 also.

What did the students think about the changes to 'A' level teaching?
Student feed-back confirmed that students were positive about working collaboratively with the new activities:

'I used to prefer working on my own but now I enjoy talking in pairs.'

'Initially it was more confident ones dominating - now there is more trust and openness and everyone takes part.'

The other key area of student feed-back referred to improvements in relationships:

'[Teachers are] Spot on! Really helpful, especially with Further Maths.'

Reference

Callender, S. (2008) Raising achievement in mathematics by enhancing the learning experience. National Teacher Research Panel Conference paper

Case study 3: Improving the quality of student mathematical conversations

Case study 3: Improving the quality of student mathematical conversations

We chose this case study as it shows how a small group of teachers working together and providing challenge to each other improved the quality of students' mathematical conversations.

What was the background to the study?

Three teachers from a secondary school near Gloucester recognised that mathematical conversations between some pairs of students were not as deep as they could be and were therefore not affecting attitudes and achievement as positively as they might. They therefore set out to improve the quality and depth of these conversations.

How did the teachers carry out the study?

The study moved through four stages:

- firstly a top set Year 11 mathematics lesson on quadratic equations was videotaped and observed;
- secondly one of the teachers tried out new strategies to encourage student talk in a Year 8 lesson on sequences and rules;
- the approach was refined and then tried out on a top set Year 8 lesson on transformations; and
- finally one of the teachers applied all they had learned to a bottom set Year 7 lesson on symmetry.

How did the teachers try to improve student conversations?

The teachers used three main approaches. Students were told to:

- explain their thinking;
- convince their partner; and
- ask their partner to convince them.

The teachers modelled particular phrases to facilitate good conversations, including:

- convince me...;
- how would you convince another student that...; and
- explain why you think that...

Students were also required to make posters displaying their explanations and examples.

What was the impact of the study?

The teachers found that it was possible to help students improve the quality of their conversations. In the last lesson the teachers were pleased to find that many of the students, even the weaker ones, gave unprompted explanations to their partners and were thinking critically about what their partners said. May students were observed using the language which the teachers had modelled.

Examples from the bottom Year 7 class included:

'It's a right angle here (pointing to a corner)'

'Just count the corners (student to partner talking about symmetry)'

'coz when you twist it, it fits two times, look (student demonstrates by turning the object)'

Many of the weaker students who were not able to articulate convincing arguments were observed effectively discussing using showing and pointing.

What did the teachers learn?
The teachers learned that there was much they could do to facilitate deeper student conversations:

- model good practice;
- plan both the activity and how students were going to talk about it;
- prepare students at different points in the lesson;
- make explaining a key part of the activity;
- include written explanations but provide writing frames and other creative ideas; and
- praise good conversations when they occur.

One teacher commented:

I think one of the main lessons we learned was that children don't naturally know how to talk to each other. We have to tell them how to do it. Even weaker children can do it, if you show them how.

Reference
Pinco, E. (2008) How can we improve the quality of students' mathematical conversations?
National Centre for Excellence in the Teaching of Mathematics grant report

Case study 4: Towards an enquiry and discussion based approach to teaching and learning mathematics

Case study 4: Towards using an enquiry and discussion based approach to teaching and learning mathematics - changing practice and beliefs
We have chosen this case study as it shows how teachers at a selective 11-18 girls' grammar school who were initially nervous about introducing a new way of working, changed their practice and beliefs as a result of introducing enquiry and discussion based ways of teaching and learning.

Background to the study
Four Year 7 classes and their teachers were involved, with the assistance of a consultant. Generally mathematics had been taught in a traditional manner, based on exposition and practice and there was concern that the Year 7 curriculum did not provide sufficient challenge or progression for many of the students, nor did it allow links to be made across topics.

The school completely re-wrote the Year 7 scheme of work for mathematics with an emphasis on enquiry-based approaches. A collaborative planning approach was adopted with teachers working in pairs. Two of the teachers planned and delivered a unit of work over a number of weeks. The pairs then discussed effectiveness of the new approach before swapping and teaching the new units again. Finally, at the end of the year, the Year 7 team met to run through the entire scheme of work, discussing which approaches had been effective and amending the written scheme of work for future use.

What new approaches were used?
In moving from a textbook-based exposition and practice model to an enquiry-based approach, the teachers involved developed open-ended tasks, providing students with opportunities to consolidate previous knowledge whilst also offering a potential challenge for more confident students.

For example when working on long multiplication:

Use the digits 1, 2, 3 and 4 once each to make two numbers. Multiply your numbers together. Which numbers give you the largest and smallest possible products?

This task enabled the teacher to identify students who had difficulty with the basic skill required while allowing others to make generalisations and to justify their reasoning.

Typically students would work individually for a short period but then be asked to discuss their work in pairs, groups or as a class. Solutions and methods could then be justified and compared.

Students were also given additional resources to stimulate learning, such as geoboards, multi-link cubes or paper-folding.

How did the project challenge teachers' practice and beliefs?
Staff involved initially had mixed feelings about moving from a familiar way of working to what was viewed as an untested approach. In reality the teachers involved found it to be an inspiring experience, benefiting both themselves and their students. Comments included:

What was surprising was the richness of the content of the work and how one piece of work could open the doors to many areas of mathematics.

I was inspired to try a new way of teaching Year 7 by having [consultant] in my Year 8 lesson last summer term, when he showed how an open-ended approach to a topic can yield results which involve students doing higher level Maths than I thought possible.

Trying an enquiry approach with some of my classes.... I found that I could very quickly assess what students did or didn't already understand, and I have been consistently surprised by the insights students have shared when asked to find their own method to solve a problem.

Wider impacts on pedagogical approach

Staff commented:

The project has helped teachers to explore new ideas of delivery and has made us reflect on our teaching styles.

I am looking for more 'experimental' ways and open-ended ideas for other year groups for example: more brainstorming, 'what if', 'what do you notice', try this with a friend', 'make up your own example'.

What impact did the project have on student learning and attitudes?
Students who experienced the new curriculum with a focus on open-ended tasks and problems became:

- more likely to work at a problem that might initially appear difficult and less likely to be discouraged by being stuck;
- more inquisitive and critical in their thinking and more likely to pose their own mathematical questions;
- more likely to produce creative and varied solutions and happier to justify them to peers; and
- more enthusiastic about learning mathematics.

My class are not fazed when faced with an open-ended question or when asked to brainstorm or explain their solutions... they feel that they can 'do' maths as they aren't limited by one particular method.

As a result of the project the enquiry approach was rolled out across the whole school. A similar complete overhaul of the Year 8 and 9 schemes of work followed.

Reference

Richards, M. (2008) Teaching and learning mathematics using an enquiry-based approach. Summary of the National Teacher Research Panel conference, 2008

Case study 5: Collaborative CPD to improve practice

Case study 5: Collaborative CPD to improve practice
We chose this case study because it shows how CPD delivered over a period of time was used to stimulate more interactive teaching, which in turn improved student discussion and engagement in mathematics.

The project set out to make mathematics more exciting and engaging by encouraging students to adopt problem solving strategies, ask questions ad explore their understanding by working with others.

What were the key features of the CPD?
Two teachers from each of seven Cornwall schools and colleges came together in the project. The project leader, the county adviser for mathematics, made a conscious decision to avoid one-off INSET training. Instead the project was based around three half-day input sessions, once per term over an academic year.

The sessions focussed on modelling and developing teaching approaches and resources. The teachers were expected to trial, evaluate and develop the methodology collaboratively when they were back at school.

The adviser gave teachers support during school visits, contributing to joint planning and team-teaching. Additional support was available through a private online web portal to enable all the teachers to share their experiences, resources, lesson plans and evaluations. Between them the teachers developed over 50 teaching resources.

What was the impact on classroom practice?
The teachers involved developed a broad range of approaches and strategies for teaching. They developed new resources to support these, such as jigsaws, treasure hunts and card sorts. Strategies included helping students understand concepts through collaboration, investigations, questioning and engagement in discussion.

The new approaches moved the teachers away from traditional teacher-led didactic approach towards one which was student-led and emphasised active engagement. Students talked about mathematics more, explored ideas together and explained concepts and solutions to each other. Teachers became enablers and supporters of learning in the following ways:

- engaging with students in more subtle and unobtrusive ways;
- being more attentive to students' conversations and discussions; and
- responding to students who were not following productive paths.

What was the impact on students?
Key impacts on students reported by the teachers involved included:

Deeper understanding of key concepts
Thinking strategies were improved and discussions led to links being made between topics, leading to greater understanding. One teacher commented:

You know that feeling you get when you have taught a lesson and you know that the students have walked away with a huge understanding of a topic - well that was the feeling I had in spades after the lesson... Not only did they know how to use binomial expansions, they still knew it three months later.

Greater motivation to learn and persistence, especially by weaker students
The new strategies engaged students who had previously seemed unmotivated by familiar and traditional ways of teaching.

Greater challenge provided for more able students

Greater opportunities to think for themselves led to more able students stretching themselves more.

Improved uptake of mathematics post-16
Most of the settings reported increased uptake in mathematics courses post-16 and improved retention rates from AS to A2 level.

What effect did this model of CPD have on the teachers?
Generally teachers spoke of the excitement they felt when students produced questions or solutions that they themselves had not thought of.

The teachers felt invigorated by their involvement with the project and unanimously expressed renewed enthusiasm for teaching mathematics.

Working with their colleagues gave the teachers the encouragement and confidence to develop and try out their own ideas, but it was important that they were willing to be observed teaching and open to new ideas.

Reference
Northern, L. (2008)
Enhancing the quality of learning & teaching in post-16 mathematics
Back to top

Further Reading

Related research

Kyriacou, C. & Goulding, M. (2006)
A systematic review of strategies to raise pupils' motivational effort in Key Stage 4 mathematics.
EPPI-centre Social Science Research Unit, London.

Mercer, N. & Sams, C. (2006)
Teaching children how to use language to solve maths problems.
Language and Education, 20(6), pp.507-528.

Swan, M. (2006) Learning GCSE mathematics through discussion: What are the effects on students? *Journal of Further and Higher Education*, 30(3), pp.229-241.

Swain, J. & Swan, M. (2007)
Thinking through mathematics research report.
NRDC

Gilmore, C. & Bryant, P. (2006) Different patterns of development in primary children's understanding of addition and subtraction. *British Journal of Educational Psychology*, 76, pp.309-331.
Introduction to the report

Fuchs, & Lynn (2004)
Helping pupils classify and tackle mathematics problems.
Journal of Educational Psychology, 96(4), pp.635-647.

Bills, C. et al (2004) 'Thinkers.' Derby: Association of Teachers of Mathematics.

Resources

A number of teaching resources for mathematics, reports on their implementation and other articles are available at:

Nrich: specialists in rich mathematics

National centre for excellence in the teaching of mathematics

Tarsia software (free to download mathematical teaching software for creating jigsaws, domino and card sort activities).

Applets and other maths resources

Swan, M. (2005)
'Improving Learning In Mathematics: challenges and strategies' .
DCSF Standards Unit Resource

Ollerton, M. (2005) *'100 Problems In Mathematics'* . Continuum International Publishing Group Ltd

Prestige, S. and Perks, P. (2001) *'Adapting and Extending Secondary Mathematics Activities: New Tasks for Old'*. David Fulton Publishers
Back to top

Appraisal

Robustness
Evidence was gathered and analysed in order to answer three research questions:

- How can we design teaching so that mathematics learning will become more effective?
- What are the effects of applying such principles to the design of teaching GCSE re-take classes in FE mathematics classrooms?
- What conceptual tools facilitate the construction of collaborative cultures in these classrooms?

The research was carried out in two phases. During the first phase lasing two years, the main researcher interviewed teachers and visited four classrooms to look at existing practices and monitor their effects. Baseline data were collected on approaches used by the teachers and common difficulties and misunderstandings by students. The researcher designed a collection of classroom discussion activities using the diagnostic teaching approach. These were trialled and tested. Lessons were observed and recorded and students' levels of understanding were monitored through pre and post testing.

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- it was considered fundamental to all mathematical work and one which lower- achieving students find hard to grasp as a general concept
- algebra was a topic that had not shown much improvement during the first phase of the project
- the teachers in phase one agreed that algebra is often taught in an unimaginative, transmission orientated way.

Relevance
Despite schools and teachers constantly looking for ways to improve the teaching and learning of mathematics, there remain groups of students who struggle to achieve in mathematics and who find particular concepts difficult to grasp. This research provides evidence that altering the style of teaching and the

classroom culture towards collaboration, discussion and student-centred practices can be beneficial to the learning of mathematics. This book is relevant for teachers of all phases and not just for practitioners in the FE sector.

The discussion and reflection approaches have been subsequently developed for use with all post-16 students, with adult learners and with 11-16 year old students in schools. The DFES initiative 'Success for All' has resulted in the development of a set of resources for teaching mathematics to all post-16 students that are based on the principles of this research.

Applicability
This research shows how collaboration and discussion in mathematics is important for learning. The study concluded that student-centred, collaborative and discussion based approaches to learning were more effective than traditional transmission methods in which the teacher explains a topic and asks students to repetitively complete exercises. The lessons were particularly successful at assisting the development of conceptual understanding of mathematics. Specific impacts included:

- improved pupil scores in algebra tests
- increases in pupil motivation and reduction in anxiety around mathematics, and
- more student-centred practices used by teachers.

The new lessons also stimulated different kinds of classroom talk, including:

- students giving more reasoned explanations
- using conflict or mistakes to improve understanding, and
- greater participation in whole class discussions.

Writing
The book provides a detailed account of the research project, but it is well-sign-posted. The book consists of two parts. The first part has three chapters focusing on the purposes, theories and metaphors of learning and teaching mathematics. Part two examines how to implement the design principles, such as the design of the research tools, the programme and its effects. The language is accessible. Despite its length, the writing is lively and the inclusion of examples and diagrams adds interest. There is also a compact summary and conclusions chapter which helps the reader to immerse themselves in the substance of the work.
Back to top